Holographic Pomeron and Primordial Viscosity

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The holographic pomeron in dipole-dipole scattering follows from the exchange of a stringy instanton with an apparent Unruh temperature and entropy that are caused by the rapidity gap χ of the collision. We show that the primordial transverse shear viscosity to transverse entropy density ratio is $\eta_\perp/s_\perp = (\pi k/\chi)^2/8\pi$ for dipole sources of N-ality k. The primordial holographic matter released by the pomeron is an ideal fluid at asymptotic rapidity χ.

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1. Introduction. Collider experiments using heavy ion produces have revealed a novel state state of hadronic matter followed by large hadronic multiplicities and strong azimuthal flow in the collision plane. The general lore of energy-momentum conservation and local thermalization through the use of hydrodynamics to describe the details of the flow and the hadronic multiplicities at thermal freeze-out points markedly to a very large initial entropy deposition with very small mean-free paths for the constituents. Prompt entropy deposition in heavy ion collisions has been attempted both at weak coupling in QCD [1–3] and strong coupling in holographic QCD [4–8].

Heavy ion collisions at large current colliders energies involve a large number of pp collisions with $\sqrt{s}$ ranging from $0.2 - 7$ TeV. pp collisions at these energies are characterized by large rapidity gaps $\chi = \ln(s/s_0)$ which are dominated by reggeon and pomeron exchanges [9, 10]. Holographic QCD in the dual limit of large N_c and strong t Hooft coupling $\lambda = g^2 N_c$, describes pomeron exchange as a close string exchange in non-critical 5 dimensions [11, 12]. The extra fifth hyperbolic dimension captures the aspect of QCD evolution in the conformal limit. An alternative description using the Virasoro-Shapiro amplitude in critical 10 dimensions for the pomeron has also been suggested in [13].

At large rapidity χ, this close string exchange is characterized by an effective Unruh temperature. This temperature is caused by the emergence of a local acceleration on the string world-sheet needed to interpolate between the receding string end-points of opposite rapidities $\pm\chi/2$. This Unruh temperature is lower than the string Hagedorn temperature. However, it is enough to excite the string tachyon in non-critical dimensions and therefore induces entropy. This stringy entropy is neither thermal nor coherent in essence. Estimates show that the entropy released is about 1/3 per dipole-dipole collisions, with about 10 dipoles per pp collisions at typical collider energies [14].

The stringy entropy released in individual pp collisions translates to a formidable prompt entropy in AA collisions under the assumption of holographic saturation [12, 14]. Most of the measured charged multiplicities at collider energies can be reasonably assessed. This entropy is released over short time scales. We now suggest that this promptly released hadronic matter made out of string bits behaves as a fluid for the higher string modes with very low primordial viscosity. The fluid is ideal at asymptotic rapidities χ.

In section 2, we argue that the transverse pomeron propagator is actually a thermal partition function for the transverse string modes with a small apparent temperature at large rapidity χ. In section 3 we detail the construction of the transverse stress tensor and use it to assess the primordial shear viscosity in linear response. Our conclusions follow.

2. Transverse Partition Function. At large rapidity χ the pomeron exchange can be viewed as a closed string exchange between twisted dipoles [11]. At large impact parameter $\mathbf{b}$, the effective string action is the Polyakov action in $D = 2 + D_\perp$ Minkowski dimensions [11, 12]

$$
\begin{aligned}
S \;=\; & \frac{\sigma_T}{2}\int_0^T d\tau \int_0^\pi d\sigma \left(-(\partial x^0)^2 + (\partial x^1)^2 + (\partial x^\perp)^2\right) \\
+ \; & \frac{E}{2}\int_0^T d\tau \left(x^1\partial_\tau x^0 - x^0 \partial_\sigma x^1\right)\Big|_{\sigma=0,\pi}, \qquad (1)
\end{aligned}
$$

with

$$E = F_{01} = \sigma_T \tanh(\chi/2) \qquad (2)$$

a longitudinal electric field along the x^1 direction. The twist angle is played by the rapidity χ and σ_T is the fundamental string tension. Throughout, $D_\perp = 3$ with $x^\perp = (x^2, x^3, x^4)$ referring to the 2 physical transverse directions and 1 holographic direction x^4. The curved character of $\mathrm{AdS}_{D_\perp}$ on the results to follow will be addressed briefly.

N-ality k dipole-dipole scattering following from (2) with a large rapidity gap χ is dominated by the exchange of a pomeron. In flat space, this exchange is captured by the transverse propagator [11, 12]

$$\mathbf{K}_T\left(\frac{ik}{2}\frac{\beta}{\mathbf{b}}\right) = e^{-\sigma_k \beta \mathbf{b}/2}\, \eta^{-D_\perp}\left(\frac{ik}{2}\frac{\beta}{\mathbf{b}}\right) \qquad (3)$$

with $\sigma_k = k\sigma_T$ and $1/\beta = \chi/2\pi\mathbf{b}$ the Unruh temperature. The contribution $S_k = \sigma_k\beta\mathbf{b}/2$ can be identified with the a semi-classical action of a stringy instanton on the world-sheet. The Dedekind eta contribution captures the transverse quantum corrections to the stringy instanton. We refer to [11, 12] for more details on this analysis.

As an effective string action (2) describes only the pomeron exchange at large impact parameter $\mathbf{b}$. Also, at large rapidities χ the Unruh temperature is smaller than the Hagedorn temperature $1/\beta_H$. Hence the domain of validity of (3): $\beta_H < \beta < \mathbf{b}$. The modular relation $\eta(ix) = \eta(i/x)/\sqrt{x}$ satisfied by the Dedekind eta function [15], allows us to unfold

$$\eta^{-D_\perp}\left(\frac{ik}{2}\frac{\beta}{\mathbf{b}}\right) = \left(\frac{k\beta}{2\mathbf{b}}\right)^{D_\perp} e^{\pi D_\perp \mathbf{b}/12k\beta} \times \prod_{n=1}^{+\infty}\left(1 - e^{-4\pi n\mathbf{b}/k\beta}\right)^{-D_\perp} \tag{4}$$

effectively trading $\beta/\mathbf{b}$ with $\mathbf{b}/\beta$ which makes the string of exponents in (4) convergent. We note that large $\mathbf{b}$ corresponds to $\sqrt{-t} \ll \sqrt{s}$ pomeron exchange kinematics.

Following standard arguments, we now define

$$\mathbf{Z}_\perp(\beta_k) \equiv \prod_{n=1}^{+\infty}\left(1 - e^{-(4\pi\mathbf{b}/k\beta)\,n}\right)^{-D_\perp} = \mathrm{Tr}\left(e^{-\beta_k\,\mathbf{L}_0}\right) \tag{5}$$

with $\beta_k = (4\pi\mathbf{b}/k\beta) = 2\chi/k$. For a string with fixed transverse end-points

$$x_\perp^i(\tau,\sigma) = \mathbf{b}^i\frac{\sigma}{\pi} + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{a_n^i}{n}\sin(n\sigma)\,e^{-in\tau}\ , \tag{6}$$

the normally ordered Virasoro generator is [16]

$$\mathbf{L}_0 = \sum_{n=1}^{\infty}\sum_{i=1}^{D_\perp} :a_{-n}^i a_n^i: \tag{7}$$

with the standard oscillator algebra $\left[a_n^i, a_m^j\right] = n\,\delta^{ij}\delta_{n+m,0}$. Using (4-7) we may recast (3) in the form of an apparent thermal trace

$$\mathbf{K}_T\left(\frac{ik}{2}\frac{\beta}{\mathbf{b}}\right) = \left(\frac{2\pi}{\beta_k}\right)^{D_\perp} e^{-\sigma_k\beta\mathbf{b}/2}\,\mathrm{Tr}\left(e^{-\beta_k\left(\mathbf{L}_0 - \frac{D_\perp}{24}\right)}\right) \tag{8}$$

We recall that the normal ordering of (7) produces the zero-point contribution of $D_\perp/24$ in (8) [16].

$\mathbf{K}_T$ through (8) obeys a diffusion equation in rapidity

$$\left(\partial_\chi + \mathbf{D}_k\left(\mathbf{M}_0^2 - \nabla_{\mathbf{b}}^2\right)\right)\mathbf{K}_T = 0 \tag{9}$$

with $\mathbf{D}_k = \alpha'/2k$ the pomeron diffusion constant and the tachyon mass

$$\mathbf{M}_0^2 = \frac{4}{\alpha'}\left(\langle\mathbf{L}_0\rangle - \frac{D_\perp}{24}\right) \tag{10}$$

The averaging in (10) is carried through

$$\begin{aligned}\langle\mathbf{L}_0\rangle &= -\frac{\partial\ln\mathbf{Z}_\perp}{\partial\beta_k} \\ &= \mathrm{Tr}\left(\mathbf{L}_0\frac{e^{-\beta_k\mathbf{L}_0}}{\mathbf{Z}_\perp}\right) = D_\perp\sum_{n=1}^{\infty}\frac{n}{e^{\beta_k n}-1}\end{aligned} \tag{11}$$

as per the diffusion equation (9). The latter encodes Gribov diffusion of the pomeron in the form of a tachyon diffusion in rapidity χ.

Curvature corrections to (9) due to the curved nature of $\mathrm{AdS}_{D_\perp}$ and therefore (8) stems from trading the flat Laplacian $\nabla_{\mathbf{b}}^2$ with its analogue in curved $\mathrm{AdS}_{D_\perp}$. The result is a diffusive equation in $\mathrm{AdS}_{D_\perp}$ with a $1/\sqrt{\lambda}$ correction to the tachyon mass [12].

Note that since $\beta_k = 2\chi/k$ is large, the mean occupation of the transverse string modes contributing to (11) is small

$$\langle : a_{-n}^i a_n^i : \rangle = \frac{n}{e^{\beta_k n}-1} \approx n\,e^{-\beta_k n} \tag{12}$$

A large rapidity gap freezes the stringy pomeron exchange to its lowest tachyonic mode.

The free energy associated to the transverse pomeron propagator can be identified with $\mathbf{F}_k = -\ln\mathbf{K}_T/\beta$ thanks to the induced Unruh temperature $1/\beta$. This translates to a pomeron entropy $\mathbf{S}_k = \beta^2\partial\mathbf{F}_k/\partial\beta$. Explicitly

$$\begin{aligned}\mathbf{S}_k = \ & D_\perp\sum_{n=1}^{\infty}\ln\left(1 + \frac{1}{e^{\beta_k n}-1}\right) \\ & + \beta_k\left(\frac{D_\perp}{12} - \langle\mathbf{L}_0\rangle\right) - \frac{D_\perp}{2}\left(1 + \ln\left(\frac{\beta_k}{2\pi}\right)\right)\end{aligned} \tag{13}$$

Again, since $\beta_k = 2\chi/k$ is large, (13) is dominated by the tachyon contribution $\mathbf{S}_k \approx D_\perp\beta_k/12$ [14]. Therefore, the released pomeron entropy per transverse area is

$$s_\perp \equiv \frac{\mathbf{S}_k}{A_\perp} \approx \frac{1}{A_\perp}\frac{\chi D_\perp}{6k} \tag{14}$$

at large rapidity. $1/\sqrt{\lambda}$ corrections to (14) have been detailed in [14].

3. Transverse shear viscosity. For the transport properties associated to the stringy modes in the transverse 2-dimensional plane to the dipole-dipole collision at large rapidity χ, the transverse string modes are dominant. The primordial shear viscosity follows from a Kubo formula whereby the ensemble averaging is carried using (11) with an apparent thermal temperature $1/\beta_k$ which is small. Explicitly

$$\eta_\perp = \lim_{\omega\to 0} \frac{A_\perp}{2\omega} \int_0^\infty d\tau e^{i\omega\tau} \mathbf{R}_{23,23}(\tau) \tag{15}$$

with the retarded response function for the transverse stress tensor $(i, j = 2, 3)$

$$\mathbf{R}_{ij,pq}(\tau) = \left\langle \left[T_\perp^{ij}(\tau), T_\perp^{pq}(0) \right] \right\rangle \tag{16}$$

$A_\perp \approx \chi\alpha'$ with $\alpha' = 1/2\pi\sigma_T$ is the typical transverse area of the diffusing string in $D_\perp$ dimensions [11, 12].

The stress tensor in D dimensions follows from (1)

$$T_D^{\mu\nu}(y) = \frac{\sigma_T}{2} \int_0^T d\tau \int_0^\pi d\sigma \, \partial x^\mu \, \partial x^\nu \, \delta_D(x - y) \tag{17}$$

We dropped the boundary contributions as they do not affect the evaluation of the transverse transport coefficient. For the transverse spatial components $\mu, \nu = i, j = 2, 3$ we identify the time $x^0 = \tau$ with the affine coordinate on the world-sheet defined with flat metric $h = (1, -1)$. Thus

$$T_\perp^{ij}(\tau) = \frac{\sigma_T}{2A_\perp} \int_0^\pi d\sigma \, \left(\dot{x}_\perp^i \, \dot{x}_\perp^j - x'^i_\perp x'^j_\perp \right) \tag{18}$$

after averaging over the spatially transverse directions $y_\perp^{2,3}$ and integrating over the longitudinal y^1 and holographic $y_\perp^4$ directions. We are only interested in the zero momentum part of (17) along the spatial directions in (16). Inserting (6) into (18) and normal ordering yield

$$T_\perp^{ij}(\tau) = \frac{1}{2A_\perp} \sum_{n\neq 0} \, : a_n^i a_n^j : \, e^{-2in\tau} \tag{19}$$

Symmetry requires

$$\mathbf{R}_{ij,pq}(\tau) = (\delta_{ip}\delta_{jq} + \delta_{iq}\delta_{jp}) \, \mathbf{R}(\tau) \tag{20}$$

Using (19) in (16) and the averaging (12) yield

$$\mathbf{R}(\tau) = \frac{-i}{A_\perp^2} \sum_{n=1}^\infty \frac{n^2}{e^{\beta_k n} - 1} \sin(2n\tau) \tag{21}$$

Inserting (20-21) into (16) and carrying the limit gives

$$\eta_\perp = \lim_{\omega\to 0} \frac{A_\perp \mathbf{R}(\omega)}{2\omega} = \frac{1}{A_\perp} \frac{\pi}{8\beta_k} \tag{22}$$

Unlike the entropy density (14), the transverse mode contributions to (22) decrease with the rapidity gap rather than scale with it. Since the transverse string area is $A_\perp \approx \chi\alpha'$, it follows that

$$\eta_\perp \approx \frac{\sigma_k}{2} \left(\frac{\pi}{2\chi} \right)^2 \tag{23}$$

which is asymptotically small.

The ratio of the primordial transverse viscosity (22) to transverse entropy (14) is independent of the way we set the transverse diffusion area $A_\perp$,

$$\frac{\eta_\perp}{s_\perp} = \frac{3\pi k^2}{8\chi^2 D_\perp} \equiv \frac{1}{8\pi} \left(\frac{\pi k}{\chi} \right)^2 \tag{24}$$

We recall that for the holographic pomeron $D_\perp = 3$. We note that the ratio jumps by 4 in trading $k = 1$ or a fundamental dipole source with $k = 2$ or an adjoint dipole source.

(24) is remarkable as it shows that the holographic pomeron matter released in dipole-dipole scattering at large rapidity χ asymptotes an ideal fluid. The transverse matter is composed of diffusive string bits that behave like an ideal fluid upon release. Of course, the transverse area $A_\perp \approx \chi\alpha'$ over which the fluid is released is only sizable at asymptotic rapidities. In AA collisions there is ample release of this matter with a cumulative prompt entropy that is comparable to the observed charged particle multiplicities in current collider experiments [14].

4. Conclusions. In holographic QCD the pomeron exchange in dipole-dipole scattering with large rapidity gap χ is described by the exchange of a non-critical string in hyperbolic $D = 5$ dimensions. The boosted longitudinal directions at large rapidity causes the longitudinal space to be Rindler with a typical Rindler acceleration of $a = \chi/\mathbf{b}$ with $\mathbf{b}$ being the dipole-dipole impact parameter. As a result the string world-sheet experiences a local Unruh temperature of about $T_U = a/2\pi$, which leads to a transverse stringy entropy of the order of $1/3$ per dipole scatterer [14].

As the Unruh temperature is smaller than the Hagedorn temperature, this primordial entropy is mostly carried by the tachyonic string mode. The transverse string modes are excited, but their contribution to the transverse entropy is sub-leading at large rapidity χ. However, the transverse string modes are the dominant contributors to the transverse energy momentum tensor and therefore their fluctuations dominate the transverse transport properties of the primordial matter released by the pomeron exchange.

The transverse pomeron propagator in the holographic limit, reduces to an apparent thermal trace over the transverse modes with a small but dimensionless apparent temperature $1/2\chi$. Transverse transport properties of the released stringy matter are then emenable to standard linear response theory with apparent thermal factors induced by the rapidity.

The transverse shear viscosity of the primordial matter released by the exchange of the pomeron over its transverse diffusive size $A_\perp \approx \chi\alpha'$ is found to be small, i.e. $\eta_\perp \approx 1/\chi^2$. Unlike the transverse entropy density which is constant over the rapidity gap thanks to the tachyon, the transverse viscosity is not. As a result, the ratio of the primordial transverse viscosity to transverse entropy per unit area (24) is found to vanish asymptotically.

The stringy matter released by the holographic pomeron in the transverse collision plane is an ideal stringy fluid at asymptotic rapidities χ. This limit evades the $1/4\pi$ lower bound [17] as it involves the exchange of a string that is in apparent rather than true thermal equilibrium. While the transverse entropy is dominated by the tachyon and scales with the rapidity gap, the viscosity is due to the transverse modes which are suppressed by the rapidity gap. The transverse modes are kinematically subdominant in bulk but dominant in transverse transport. This dichotomy is the essence of the dynamical rather than thermal calculation we have detailed, which may well be the lore at current collider energies in the primordial stage.

Adjoint dipole sources with $k = 2$ are expected to be released in central AA collisions in current collider energies whenever the number of nucleon participants exceed 10 [14]. Primordial AA matter is about 4 times more viscous than pp and pA matter for central collisions. In the rapidity range $\chi \approx k\pi$ the primordial ratio (24) is about $1/8\pi$. This rapidity range is the one currently accessible by current ultra-relativistic heavy ion colliders.

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